

Multiplicity of Mental Spaces

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Erroneous and Precise Spatial Actions

When asked the direction between Philadelphia and Rome, most people err. They say that Philadelphia is north of Rome when in fact, it is south of Rome. This cannot be dismissed as the weather, because when asked the direction between Boston and Rio, a majority of people erroneously say that Boston is east of Rio. Nor are these errors a simple consequence of randomness, nor of ignorance of geography. Rather, they are systematic and predictable outcomes of the way spatial information is organized in the mind. Contrast these errors with the precise artistry of violin playing or basketball or surgery, or wending one's way through a crowd. How is it that one set of spatial behaviors is predictably erroneous and the other predictably precise? That one set of spatial behaviors appears blunt and irrational and the other delicate and tuned?

How, then, to reconcile spatial behaviors that are finely-tuned and precise with those that are clumsy and erroneous? A closer look at the finely-tuned, precise ones reveals that these behaviors are repeated, indeed practiced, in structured environments that provide feedback and that are replete with cues that guide and support behavior. Sheer repetition in supportive environments provide these perceptual-motor performances the benefits of selection, by evolution or by learning. Behavior that yields good outcomes is selected and repeated; not so behavior whose outcomes are not favorable. Rich environments provide cues for the behaviors more likely to yield favorable outcomes. In short, through practice with feedback, actions become precisely tuned. By contrast, spatial behaviors that are infrequent and based on hypothetical rather than experienced environments, notably judgements, do not enjoy the benefits of either repetition with feedback or of supportive environmental cues. Instead of being situated in real environments, they are mediated by cognitive representations of environments that are local, ad hoc, and schematic. Although the cognitive structures constructed to represent environments and to enable judgements on them are based on qualities of the perceptual world, the fact that they are schematic introduces systematic error and the fact that they are ad hoc and local means there is no guarantee of global coherence, introducing the possibility of inconsistency.

Ad hoc, local, schematic cognitive structures are invoked when there are no means, either by the tools of the mind or by the tools of the world, of producing complete representations on which appropriate and accurate calculations can be performed. Given the constraints of human information processing, especially limited capacity working memory, the use of ad hoc, local, schematic cognitive structures can provide relatively quick and efficient estimations. Some of the errors inevitably introduced by these heuristics may be corrected by constraints in the field or by constraints from other estimates or modes of estimation that may be independent. Other errors, however, may go undetected. Some may lead to unfortunate consequences.

What cognitive mechanisms are responsible for spatial behaviors that are precise and tuned and what cognitive mechanisms underly spatial behaviors that are schematic and biased? Precise spatial behaviors, from infants reaching for toys to individuals finding their ways home to couples dancing in consort develop with feedback from extensive practice in specific environments. Spatial skills such as these are situated, that is, executed in environments replete with cues that support, constrain, and guide accurate performance. The specific instrument of the musician, the bodies and tools of the surgeon, the landmarks and layout of the navigator's environment serve to both constrain the spatial behaviors and to cue them. . Initial attempts at reaching or wayfinding or dancing are clumsy and fumbling. With practice, the behaviors become accurate perceptual-motor skills that can be enacted automatically, with little demand on attention.

Contrast these cognitive mechanisms with those underlying answer questions or filling requests such as these. "You're standing in the Piazza Navona, facing Bernini's Fountain of the Four Rivers, what's to your left?" "How far is Philadelphia from Pittsburgh?" "From New York City?" "Can you draw me a map to get to your house?." Why is it that such questions are answered from ad hoc, local, schematic mental constructions? As usual in biological systems, there is more than one reason. One factor, already mentioned, yielding schematic mental representations is constraints on working memory. Another is the nature of long-term memory representations. Questions such as those above are hypothetical at the time they are asked; that is, the responder may not be in the environment asked about. Or the questions may be about environments too large to be experienced at once. In order to answer the question, the responder must construct a mental representation of the environment, and then make a mental judgement on the mental representation. Thus the the question must be answered, the judgment must be made on a mental construction of the environment, not on the actual environment. Both the mental construction and the judgement are made in working memory which has a limited capacity (e. g., Baddeley, 1990). Constructing a schematized version of the environment tailor-made for the particular query is one way of conserving working memory space.

There is yet another factor encouraging mental constructions of environments that are ad hoc, local, and schematic. Unlike maps people can consult on-line or on the shelves, there may be no ready-made mental representation of the information demanded by the task. This means that the mental representation must be created on the fly from information in long-term memory. The information about environments in long-term memory does not resemble in either structure or format a map such as those developed by cartographers for a variety of purposes. "Cognitive maps" – that is, whatever mental information is used to make geographic judgments – are notoriously inaccurate and incomplete. What's more, they seem to have a variety of formats consisting in part from memory for maps that may have been studied, in part from memory for experiences in environment,s in part from memory for descriptions of environments. Because of the diversity of formats that have no natural or standard means of integration for the mental representations of spaces, "cognitive collage" has been suggested as a more apt metaphor than "cognitive map" (Tversky, 1993). In order to integrate these different pieces of information, especially when they are in different formats, a common structure is needed. A common structure can be provided in several ways, for example, by landmarks or features common across pieces of information coded by a common reference system. A pictorial memory of

an environment including the city hall and the post office could be combined with a verbal memory of the route instructions from the post office to the bank by the common feature of the post office to allow construction of a route from city hall to the bank. Or a memory of a subway map may be integrated with memories of the neighborhoods around subway stops to allow estimates of directions between landmarks at different subway stops. However the bits of information are combined, they are combined roughly and schematically.

As noted, answers to queries about the spatial world are prone to error because the answers are determined in working memory, which has capacity constraints. When answers are not known, they are determined by mentally constructing an ad hoc representation of the space and performing a computation on that representation. However, even if working memory capacity were augmented by using an external representation for computation (e.g., Donald, 1991; Norman, 1993, Tversky, in press), the likelihood of systematic error remains because the sketch maps would be produced schematically combining incomplete fragments of information in different formats. In fact, sketches also demonstrate the sorts of errors and biases that will be described in detail below.

The solution to both the constraints of working memory and the nature of long-term memory representations of environments is mental structures that are tailor-made for the query at hand, constructed from different pieces and kinds of information, selecting only the information relevant to the query, integrating it through a common schematic framework. What's more, the common schematic framework varies with the type of query. Let us now turn to two case studies illustrating this process, one for the space around the body, the other for the space of navigation. Each of these spaces is conceived of differently, in ways that depend on how the space is perceived and used in daily life, that is, on perceptual and functional aspects of the space. Thus, space, as conceived of by ordinary people as they interact with it differs from space as conceived of by geometers or physicists. For the latter, these different spaces are coherent and conceptualized in a unitary fashion.

The analysis of the two spaces will illustrate these features of spatial cognition: that different spaces are conceived of differently, that the way the spaces are conceived depends on perceptual and behavioral experience with them, that the spaces are conceived of schematically, and that the schematization yields bias and distortion.

The Space around the Body

As we move about in the world, we remain aware of the things around us, even the things that are no longer in view. As we move forward, what was formerly ahead of us is now behind us; as we turn, what was formerly ahead of us is now to the side. We are aware of the surrounding scene even when we do not actually perceive it. After all, our range of perception is small, and for the most part, oriented forward. As long as things don't move, we can reach to the side or move backwards without looking and without erring. This awareness of the surrounding situation under movement extends beyond environments acquired by perception and navigation to hypothetical environments acquired only by description (Franklin and Tversky, 1990; also, Bryant, Franklin and Tversky, 1992; Bryant,

Tversky, and Lanca, in press). Imagined environments and imagined navigation in imagined environments capitalize on the extensive experience people have in navigating real environments, on the mental structures people construct in order to maintain awareness of surroundings. In several dozen experiments, participants studied descriptions of observers (addressed as "you") in environments such as a museum or hotel lobby, surrounded by objects, such as chandeliers and fountains, to their front, back, head, feet, left, or right. They were then told that they have turned to face a object formerly at their sides, back, head, or feet and queried for the objects currently at back, front, left, right, head, and feet. Participants were able to answer these questions quickly and accurately, indicating that people are able to construct mental models of space from descriptions and quickly update their hypothetical positions in them. Interestingly, although care was taken to make sure that no part of the environment was privileged and that objects were randomly assigned to positions around the body, times to retrieve the objects depended on the direction queried as well as the orientation of the observer, upright or reclining. That is, certain directions from the body were retrieved faster than others for reasons having nothing to do with the particular content of the space or objects in it.

The biases in retrieval times could be accounted for by the Spatial Framework Theory (Franklin and Tversky, 1990). Human bodies have three natural axes, formed by head and feet, front and back, and left and right. The head/feet and front/back axes are asymmetric, but the left/right axis is relatively symmetric. The asymmetries make the poles of the axes more discriminable. The world, too, has three natural axes, only one of which is asymmetric, the up-down axis formed by gravity. When a person is upright, the up/down axis of the world coincides with the head/feet axis of the body, enhancing the discriminability of that axis. According to the Spatial Framework Theory, retrievability is enhanced by discriminability so that times to retrieve objects at head and feet should be fastest, because of the confluence of asymmetry of the body and asymmetry of the world, followed by times to front and back, with an asymmetric body axis. Times to retrieve objects at left and right should be slowest as that body axis is for the most part symmetric. This pattern of retrieval times in fact emerged in many studies. The situation of the reclining observer is different. Here, the observer rolls from front to back to side so that no axis of the body is aligned with the asymmetric axis of the world. When just the body is considered, the front/back axis is more salient than the head/feet as the front/back axis separates the world that can be seen and manipulated from the world cannot be readily seen or manipulated. Again, this analysis of asymmetry, hence discriminability, conforms to the pattern of retrieval times obtained for the reclining observer: retrieval times to front and back were fastest, followed by times to head/feet and then left/right.

Many variations of these situations have been studied, with consequent variations in the patterns of retrieval times. Some of the situations studied include those where the viewpoint was outside the scene looking on rather than surrounded by objects, those where there were two people each surrounded by a different objects, and those where the room was described as rotating rather than the observer (in order: Bryant, et al., 1992; Franklin, et al., 1992; Tversky, Kim, and Cohen, 1999). The environments have been conveyed by description, by actual experience, by models, and by diagrams (in order: Franklin and Tversky, 1990; Bryant, et al., in press; Bryant and Tversky, 1999). In all cases where directions or objects were retrieved from memory, the pattern of retrieval times fit

the spatial framework theory, indicating that the memory representations established from experience and from language are functionally similar.

The point of elaborating this example is to illustrate how simple spatial judgements are made in memory. A schematic mental model of the relevant portion of the environment is constructed in working memory, and the judgement made on that representation. The spatial mental model constructed is not arbitrary; rather, it reflects people's conceptions of the spatial world, in this case, the human body and surrounding space. As we have seen, it is those conceptions of the world that led to the patterns of retrieval times. These conceptions of space derive from people's perceptual and behavioral experience in the spatial world. Thus, the schematic mental structures used to represent the world surrounding the body come to yield judgements that are biased. The biases are not meant to be interpreted as irrational in themselves, unless, of course, there were good general reasons why other directions should be privileged in retrieval time. Or, they would be irrational if the schematization led to systematic error. The next example establishes that.

The Space of Navigation

As we have seen, the space around the body is relatively small, in reach of arms or eyes, and conceived of in three dimensions. In contrast, the space of navigation is potentially vast and is conceived of primarily in two dimensions, map-like. Unlike contemporary geographic information systems, we do not seem to carry in our minds a map of the globe that can be retrieved from particular perspectives, with requisite detail, at specific degrees of resolution. We do not seem to have such cognitive maps even for environments and maps that we know well. Rather, we seem to retrieve whatever information we can to address the problem at hand. As observed earlier, that information may be fragmented, partial, and multimodal. For actual judgements, the disparate pieces may be linked together through a schematic structure that combines common elements, framework, or perspective.

Systematic errors give insight into the nature of the schematic structures used to link disparate spatial information. Elements or geographic entities such as buildings, roads, cities, and countries, serve as reference objects for each other. Global reference frames such as the surrounding environment or the cardinal directions also serve as referents for the location and direction of geographic entities (see Tversky, 1981; 1992).

Evidence for the use of geographic entities as reference objects comes from judgements where spatial relations to reference objects are exaggerated or simplified. For example, when people are asked to judge which map of the Americas is correct, a veridical map or one that has been altered so that South America is more directly south of North America, a significant majority pick the incorrect map. Likewise, people prefer a world map which has been altered so that the United States and Europe and South American and Africa are more aligned east and west over the real map. These errors can be accounted for by Gestalt principles of perceptual grouping. In order to perceive and comprehend scenes, people group related figures, for example in maps, geographical entities such as continents or countries or cities or roads. In memory, the spatial relations among these figures are exaggerated or simplified, so that large comparable entities like North and

South America are remembered as more aligned, in this case, north and south, than they actually are. Similar errors of alignment occur for judgements of the spatial relations among cities in these continents, for example, thinking that Philadelphia is north of Rome or Boston west of Rio. Alignment errors also occur for artificial maps and even for meaningless blobs.

Evidence for the use of global reference frames as anchors for localizing objects also comes from systematic errors of judgement. For example, to most, South America appears tilted with respect to the cardinal directions. When people are asked to place a cut-out of South America as accurately as possible with respect to north-south east-west coordinates, they err in the direction of rotating South America toward upright. As for errors of alignment, errors of rotation appear in other contexts. The San Francisco Bay area is also tilted with respect to the cardinal directions and uprighted in memory so that people erroneously indicate that Berkeley is east of Stanford, and Stanford east of Santa Cruz. Similar errors occur for other actual environments, for artificial maps, and for meaningless blobs. Like alignment, this error is related to Gestalt perceptual organizing principles, in this case, common fate. The framework or natural axes of a figure are organized with respect to the framework or natural axes of the environment encompassing the figure. In memory, the natural axes of figure and ground are rotated closer in correspondence. Thus, using other entities and global axes as references for judging location and direction distorts those judgements in the direction of the reference objects and frames.

Errors of alignment and rotation are among a large number of systematic errors revealed in judgements of direction, location, and distance (see Tversky, 1992, 1993, 2000a, and 2000b for reviews). One of the more striking errors is landmark asymmetry: estimates of distances to a landmark are smaller than estimates of distances from a landmark to an ordinary building (Sadalla, Burroughs, and Staplin, 1980). This, of course, violates euclidean metrics. Together, the errors give insight into how mental representations of environments are constructed for the purposes of particular judgements. Geographic entities, such as streets, cities, or continents, or other spatial entities are organized with respect to each other and with respect to a global framework. Apparently, only the bare essentials, the schematic minima, that are needed for making the judgement are evoked. When more elements are evoked, these introduce independent constraints that serve to cancel each other and decrease errors (Baird, 1979; Baird, Merrill, and Tannebaum, 1979). But when the judgements are done as they typically are, piecemeal, there is no guarantee that the bits and pieces of environments evoked to make the judgements are coherent.

These errors, alignment, rotation, and others, are a consequence of the perceptual organizing principles used to schematize the environment. If this were the only information people could use in navigation and in making spatial judgements, then people would err frequently. It is easy to conceive of situations where the errors would be consequential, for example, in deciding what direction to take to escape danger. Why is spatial information organized using principles that result in systematic error? Why didn't evolution and learning produce a system with greater accuracy and less bias?

Of course these are unanswerable questions. One can only surmise the reasons. Often, these sorts of judgements are unique to a particular time and specific place. They are not likely to be repeated, especially repeated with correction, and if they are, the corrections

are made for the particular environment rather than for the general principles, the underlying system producing schematizations. Moreover, there may be natural corrections for error built into the environments. Let us now turn to route directions to see how that might happen.

Route Maps and Directions

A case that illustrates the interplay of schematic representations with corrective environments is that of route instructions. Route instructions, whether instantiated in depictions or descriptions, have a similar structure (for the structure of descriptions, see Denis, 1997; for extensions to depictions and descriptions, see Tversky and Lee, 1998, 1999). They are strings of segments containing four components: start points, reorientations, progressions, and end points. Each of these components is schematized (Tversky and Lee, 1998, 1999). This is not surprising for descriptions, as language itself is schematic. What is more surprising is that sketch maps are schematic, often even categorical, as sketch maps are potentially analog. For example, in sketch maps, paths tend to be drawn as straight or curved. The segments that are drawn as straight and those that are drawn as curved parallel the language used to direct travellers along them. For straight paths, directions use “go down.” For curved paths, directions use “follow around.” Intersections are typically sketched as at right angles. Where they deviate from right angles in sketches does not correspond to where they deviate from right angles in environments. Distances are notoriously incorrect in sketch maps, and vague in verbal directions. Thus, sketch maps and route directions seem to be constructed from the same underlying information, despite differences in format. Even though they are schematic, directions can be successful, even in such difficult environments as Venice (Denis, Pazzaglia, Cornoldi, and Bertolo, 1999). Why is that? Presumably because the environment itself removes the ambiguity and corrects the errors of the sketch maps and directions. For example, exact distance does not matter, as the point of change in direction, the reorientation, will be indicated by the appropriate landmark. Similarly, people may remember an intersection as at right angles, a typical consequence of alignment and rotation. If the intersection is in fact at some other angle, the traveler has no choice but to go at that angle, rather than the angle remembered. In fact, the traveler may not even notice that the turn taken is not a right-angle turn. Schematic knowledge, then, is often sufficient for successful navigation.

Why Systematic Errors Survive

The mechanisms that generate schematic representations may remain despite the fact that they can also generate systematic error. The reasons for this derive both from characteristics of the human mind and from the nature of human interaction in the world. The mind does not seem able to generate richer representations of environments both because long-term memory does not seem to contain them and because working memory is too limited to construct them. It doesn't seem to make sense for the mind to store each and every encounter with the world in great detail. Remembering the details of a particular route

traveled to get somewhere doesn't help on the return home, or on another route to the same destination. Remembering the schematic structure of the route does. The same argument can be made for other kinds of information, for example, remembering faces. In fact, S. the mnemonist studied by Luria, had was haunted by details of episodes, which he could not forget. As a consequence, he had difficulty recognizing patterns and generalities (Luria, 1968). For example, he was poor at recognizing faces, confused by each encounter with an individual. Thus, there are real benefits for schematic representations, not just costs. Interactions with the world do not seem to correct the mechanisms producing schematization because errors may go undetected either because of lack of feedback or because the structure of the environment corrects them. If error is detected, the correction may be specific to the context, and not to the underlying mechanisms. Readers of this paper now know that Philadelphia is south of Rome, but knowing that will probably not prevent believing that Algiers is south of Los Angeles. College students who heard an entire lecture on systematic errors in cognitive maps, including errors of alignment and rotation, persisted in making those errors even when tested immediately after the lecture.

Thus, a case can be made for rational reasons for (seemingly) irrational behavior. In some sense, it is not rational to have cognitive mechanisms guaranteed to produce error. Framed differently, however, these very same mechanisms seem reasonable, in fact, a good solution to a hard problem. Systematic errors in cognitive maps are not the only systematic errors of judgement or behavior documented in people. Nor is the analysis described here the only way to make seemingly unreasonable behavior appear reasonable. Many errors of judgement occur because the human mind is asked to measure things it has no way of measuring. Instead, it substitutes close mechanisms that it does have, for example, using availability, or the ease to which things come to mind, as a heuristic for measuring frequency (Tversky and Kahneman, 1983). Since there are other factors besides frequency that cause things to come to mind, frequency judgements based on availability will be biased by these factors.

This tension, between apparently irrational behavior on the one hand and theories of selection on the other exists not only in domains of individual behavior, where selection is by evolution as well as learning, but also in other domains where selection operates, notably survival in biological, economic, and political situations. In those arenas, too, social and biological scientists delight in discovering examples of irrationality, behaviors that are self-defeating or that their instigators would disavow. For each of these, scientists also delight in construing them as reasonable, if viewed differently. Where does this leave rationality?

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