1. Introduction: How can we think about thinking?

How does thought happen? Perhaps because thought is often expressed in language, it is often thought that thought happens in language. Yet it is elementary that thought occurs in the brain, activities of neurons, in concert with the body and the world, again through the actions of neurons. The brain doesn’t speak a natural language, not within itself and not to the body. Nevertheless, language provides a compelling analogy for thought. One way to characterize thought is as consisting of elements—ideas of all kinds—and relations among them. Much like nouns and predicates, the core of propositions. But saying that thought can be regarded as structured similarly to language is structured is not to say that thought occurs in language. Both language and thought can be characterized on various levels; at some levels, there are similarities, at others, differences. Imagistic reasoning, which is usually contrasted with linguistic reasoning (e.g., Kosslyn, 1994; Johnson-Laird, 1983; Tversky, 2000), can also be characterized in terms of elements and relations (e.g., Tversky, 2001). How, then, do imagistic and linguistic thought differ? Thought is internal, invisible, opaque. Subjective analysis of thought is dizzying; objective analysis is tricky.

An easier task, perhaps, is to characterize manifestations of thought that are linguistic or imagistic. This is not to claim that internal linguistic and imagistic reasoning are the same as external manifestations of them. External manifestations of imagistic thought include maps, charts, diagrams, and even gestures. External manifestations of linguistic thought include written and spoken words. Significantly, external representations in the wild, as used by people, are rarely purely one or the other. Spoken language is accompanied by gesture and intonation; written language by punctuation and paragraphing. Similarly, maps and graphs are accompanied by symbols and legends. Language has been the focus of many treatises; the focus here is graphics, diagrams, charts, expressions of visuospatial reasoning. As for external representations of linguistic thought, external representations of imagistic thought can be characterized on many levels. Although at one level, both can be characterized as elements and relations, a crucial difference is that for imagistic representations, meanings can be carried by resemblance of elements and spatial proximity among them. This of course suggests that internal imagistic reasoning may also depend on resemblance, spatial proximity, and spatial transformations, and there is ample evidence that it does (e.g., Kosslyn, 1994;

2. Thought in External Images

Humanity has created external images since its’ beginnings. Cave paintings, petroglyphs, wood carvings, weavings, baskets, and stelae bear witness, and bear witness to the many roles images serve humanity, sacred and religious, historical and political, expressive and informative, artistic and playful, inferential and creative. Whatever their roles, images created by people share certain features. For one thing, they simplify or schematize the information. Animals depicted in cave paintings and petroglyphs are often mere outlines, sacred symbols have symmetries. Maps, whether on stone or wood or paper, use lines for paths and crossed lines for intersections, and do not respect exact distances or direction (Tversky and Lee, 1998, 1999). What features are included and what are left out, and how features are simplified is systematic, not random, and reflects how people think about the things imaged (Tversky, 2001).

In simplifying and schematizing, images and other external representations of thought are like language, which also simplifies and schematizes. Of course there are many differences between image-like and language-like representations of thought, and specifying these differences—and similarities—is a long and controversial discussion, not to be settled here. One difference that has been observed repeatedly (though controversially, e.g., Goodman, 1968) is resemblance, that external images of elements have physical similarities to what they represent. Supporters of this claim remind the skeptics that children spontaneously recognize objects from line drawings of them (Hochberg and Brooks, 1962) and that adults who have never seen maps nevertheless produce maps of their regions that resemble each other and resemble maps sketched by novices all over the world (e.g., Harley and Woodward, 1987, 1992; Woodward & Lewis, 1994, 1998).

Schematization is one reason external representations, whether image-like or language-like, are so useful. The schematization of external representations (and of language!) not only reflects people’s cognition of the thing represented, it also communicates essences to others by calling attention to certain features of the world and ignoring others. Presumably, those features that are included are the features that are important in human activities. Schematization is not without costs. One price of schematization is ambiguity; general concepts refer to many possible entities or ideas, and the actual reference or predicate may not be clear. Diplomacy, of course, relies on exactly these ambiguities. And so does reasoning. Schematization allows generalization; allows assertions, whether depicted or described, about a class of things, not a specific thing.

External representations are cognitive tools, Although other species have created tools to augment their physical well-being, such as tools for digging or reaching, it does not appear that other species have created tools to augment their mental well-being. External representations serve their creators and others in a broad range of functions. They record information, preserving it for cultures and for individuals. They serve as a
viewable and reviewable platform for thinking and reasoning, relieving the burdens of working memory. Because they are public, they can be used simultaneously by groups to refer, to reason, to revise, and to assure common understanding. By metaphor, they express abstract as well as concrete ideas, and they allow inference and leaps of imagination based on proximity, direction, distance, similarity.

Especially for reasoning, inference, and leaps of imagination, diagrammatic external representations are more productive than linguistic ones. Their productivity derives in part from the correspondences of elements in diagrammatic space to elements in conceptual space, and of spatial relations to conceptual relations. This transforms abstract problems, such as comparing wealth of countries or understanding conduct of systems, into spatial ones. People have extensive experience solving spatial problems in the course of moving about in the world, an activity nearly impossible to avoid. External representations also transform internal memory and information processing into external memory and information processing, relieving the severe constraints of working memory. Spatial organization can supplement and augment reasoning from written language. For example, propositional arguments were written in essentially a tabular form in ancient Greece, a form that, it has been argued, that facilitated the development of syllogistic reasoning (Netz, 1999). In recording, revealing, and revising thoughts, external representations do not function alone, just as standard language does not serve thinking alone. Diagrams and language refer to things other than themselves, and those things can be thought about, manipulated by the mind in the many ways that minds can manipulate. More than that, especially diagrams can be used in conjunction with thought and written or spoken language, with all its’ breadth. For diagrams, gestures are a particularly effective in augmenting thought. (e.g., Heiser, Tversky, and Silverman, 2004). Some examples, drawn from our research, may clarify and illustrate. First, let us consider some ways diagrams schematize and reflect thought.

2.1. Schematization: Route maps. It is appropriate to begin with maps, as they are ancient, familiar, varied, and a popular metaphor. Maps preserve different information depending on their function. Topological maps aid hikers just as highway maps aid drivers. Useful tourist maps mix perspectives, presenting streets as if viewed from above and landmarks as if viewed frontally, allowing tourists to find their ways to the landmarks, and recognize them when they arrive. Because maps, like spoken language, have been used in communities across time and space, they have undergone millennia of informal user testing. This grants most of us expertise in map design for certain uses, notably to conveying routes, so that clues to qualities of effective maps can be found in people’s spontaneous productions of sketch maps. To reveal qualities of effective route maps, we asked hungry students to provide either sketch maps or directions to a nearby fast food restaurant (Tversky and Lee, 1998, 1999). Of course there was variability in both the maps and the directions, some were crisp and sharp, others detailed or sloppy. Despite these differences, there was considerable similarity in the elements and structures of both maps and directions. Previous work by Denis (1997) served as a model. An analysis of a large corpus of route directions revealed that they consisted of strings of segments consisting of four components: start point, reorientation, progression, end point.
Both the route maps and the route directions produced by the students could be decomposed into these components. Language is categorical, and although maps could be analog, they schematized information in ways that paralleled language. Turns were represented as +’s or T’s or L’s; the exact turn was not specified. Similarly, language indicated turns, but not the exact angle, as in “take a,” “make a,” or “turn.” Exact distances were not specified in either maps or language. In fact, the length of the description as well as the size in the map corresponded to the number of turns more than to the actual distance. Curviness of path was dichotomized in both language and in depictions. Where paths were curved, directions said “follow around;” where paths were straight, directions said “go down.” Thus elements of description and elements of depiction were parallel, and they expressed the components of routes, start and end points (intersection) and paths. The similarity and systematicity of the elements suggested that a relatively small number of elements, either verbal or pictorial, could serve to produce a large number of routes. In fact, students provided with verbal or depictive tool kits consisting of these elements were able to use them to produce a variety of routes, short and long, simple and complex, without adding other elements.

Sketch maps and route directions alike express the underlying cognitive structure of routes: a sequence of paths and nodes, actions at landmarks. They consist of parallel elements conveying similar meanings, paths and turns primary among them, elements that can be combined according to rules to represent a variety of routes. The pragmatics of route depictions and descriptions differ. For example, route directions can be discontinuous; they can, and frequently do, omit either the end point of one segment or the start point of the subsequent one, as they are the same and can be inferred. Depictions must be, and in practice, are continuous.

The elements of route maps, paths and intersections, lines and connections of lines, are readily interpreted and readily produced. This is, we propose, because the Gestalt or geometric properties of the elements suggest their meanings in context. Lines are paths, they connect. Intersections of lines suggest intersections of paths. Lines and combinations of lines serve other diagrams as well. In circuit diagrams, lines and connections are for transport of electricity, not for transport of humanity. In decision trees and flow charts, the lines connect decision points for thoughts or information rather than for travel. Let us now turn to other examples of graphic meanings.

2.2. Graphic Elements: Lines and Bars. For route maps, lines indicate connections between two points. Metaphorically, lines do the same in graphs; they indicate a connection or relationship between two (or more) points. Bars, by contrast, are containers; they separate. Thus, a bar between two points, A and B, suggests that A and B are related on the same dimension, but differ in values on that dimension. Bars for A and B suggest that there are A’s and there are B’s and that they are different. If so, the same data presented as lines or bars should be interpreted differently. Specifically, line graphs should elicit trend descriptions, such as “there’s a rising relation between A and B,” and bar graphs should elicit discrete comparisons, such as “the B’s are higher than the A’s.” In fact, this is exactly what Zacks and I (1999) found when we asked students...
to interpret graphs, both for blank graphs and for graphs whose axes were specified by continuous or discrete variables, height of 10 and 12 year olds or height of women and men. The effects of the graphic form overrode the structure underlying the data, so that some informants said of line graphs of height of women and men, “as you get more male, you get taller.” The effects of forms of graphic elements on meaning appeared in production as well as comprehension. When asked to produce graphs of trends, students produced more line graphs and when asked to produce graphs of discrete relations, students produced more bar graphs.

2.3. Graphic Elements: Arrows. Arrows are asymmetric lines, so they indicate asymmetric relationships. A case can be made that arrows naturally indicate directionality. Arrows for shooting point in the direction that they travel as the channels cut by descending water point in the direction of the flow of the water. Arrows, then, should suggest asymmetric relations, such as order. To test this, Heiser and I (submitted; Tversky, Zacks, Lee, and Heiser, 2000) asked students to interpret mechanical diagrams of a car brake, bicycle pump, or pulley system. Some students interpreted a diagram without arrows, others interpreted a diagram with arrows. The interpretations of the diagrams without arrows were predominantly structural descriptions, descriptions of the spatial relations among the parts of the mechanical systems. The interpretations of the diagrams with arrows were quite different. Arrows elicited functional descriptions; the interpretations described the actions and outcomes of the systems from start to finish. Production of diagrams mirrored these effects. When given structural descriptions and asked to produce diagrams, students produced diagrams without arrows. When given functional descriptions and asked to produce diagrams, students produced diagrams with arrows.

Arrows appear in diagrams with many different meanings. They point and label; they indicate direction and manner of motion, they convey sequence, causality, and outcomes. A recent survey of diagrams in scientific textbooks revealed these and other uses of arrows. Unfortunately, the different senses of arrows were not always disambiguated.

2.4. Reasoning with Diagrams: Representing Change in Thought. Diagrams are static images, but reasoning is dynamic. In reasoning, initial information gets transformed in some way, yielding conclusions, inferences, insights. Arrows, then, serve a crucial role in diagrams, they turn static into dynamic, they indicate transformations of thought. Arrows, of course, are not the only way to externalize changes in thought: successive diagrams showing successive steps in reasoning are another common and readily comprehended method for showing change. Successive stills have a further advantage; they can segment the changes at the critical points of change, of transformation. This is in contrast with typical animated diagrams, which present time directly proportional to time.

2.5. Meanings of Elements. Diagrams use simple schematic forms to convey meanings, lines, arrows, and bars among them. The meanings of these forms are suggested, perhaps constrained, by their Gestalt or geometric properties and hence are readily understood in context. The comparable words, relationship, line, container, are also ambiguous and
require context to clarify their meanings. Elements in diagrams can convey meanings that are readily inferred in context in other ways. Iconicity, for example, is exploited in many diagrams. Consider signage for highways, leaping deer or falling rocks. “Figures of depiction” where associated images serve to convey meaning, are also common in diagrams, for example, scissors, file folders, and trash cans in computers.

2.6. **Spatial Relations.** The elements of diagrams are not usually randomly distributed. The spatial relations among them are constrained, and the constraints are typically meaningful. The basic metaphor is proximity: near things are more related than far things. Proximity may be meaningful at different levels of information. At the categorical level, space can be used to separate groups. For example, the spaces between words and the spaces between paragraphs serve to separate ideas. At the ordinal level, space reflects orders of things, and at the interval level, space reflects both relative distances between ordered things. Spontaneous sketch maps, we noted, could reflect interval, but typically do not.

2.7. **Children’s Spontaneous use of Space in Diagrams.** Diagrams produced spontaneously by children use proximity in diagrams to convey proximity on other dimensions. We asked children as young as four years old from several cultures to place stickers on square sheets of paper to represent concepts of space, time, quantity, and preference (Tversky, Kugelmass, and Winter, 1991). Examples include the times for breakfast, lunch, and dinner, quantities of candy in a hand, a bag, and a shelf, preferences for disliked, liked, and loved TV shows. Most of even the youngest children put the stickers on a line, indicating that they thought of them as ordered on a dimension. Only the older children, 10-12 years, mapped interval, for example, breakfast, morning snack, and dinner. There was consensus across children (and adults) in communities using left-to-right and right-to-left alphabets on directions of increases for concepts of quantity and preference. Children and adults mapped increases onto left-to-right, right-to-left and down-to-up equally often; however, they avoided mapping increases from up to down.

2.8. **Sketches in Design.** Architects typically sketch in thinking through their designs. This process has been likened to a conversation (e.g., Schon, 1983) in which architects put design thoughts on paper, examine them, and then revise them. Quite frequently, architects sketch particular ideas, elements and spatial relations among them, but in examining their sketches, they discover new aspects of the design, for example, similarities across elements, new configurations, and relations among configurations. These aspects of design were not intended in the sketch, yet they promote the design. Such unintended discoveries are richer and more common for experienced architects than for novice architects, suggesting that repeated interactions in sketching and interpreting sketches yields skills in visual inference (Suwa and Tversky, 2001; Suwa, Tversky, Gero, and Purcell, 2001). Quite frequently, the unintended discoveries lead to new design ideas; similarly, new design ideas lead to new sketches and new discoveries. For experienced architects, the dialogue with their sketches is productive, an essential aspect of the design process.
There are other examples of thought expressed in and facilitated by external images, but these should serve to illustrate the breadth of concepts that can be conveyed and the ways they are conveyed, using elements and spatial relations in diagrammatic space to map elements and relations in internal mental representations. This is not to imply that external representations are the same as internal ones. One advantage of external representations is that they can exceed capacity of working memory. Another is that they allow and often demand completeness and consistency, by making omissions and inconsistencies visible. A third is that they promote inferences and unintended discoveries. Yet, as will be seen next, internal images appear to serve thought as well.

3. Thought in Internal Images

Classic experiments in cognition have shown that people can mentally rotate and scan images (e. g., Kosslyn, 1981; Shepard and Cooper, 1982). However, in these experiments, participants were presented with visual stimuli and in some cases, trained in the mental transformations. Thinking in imagery is better exemplified in tasks that are purely verbal. If input and output are verbal, how can we know that the thinking is imaginal or spatial or pictorial? This is, of course, the central problem for empirical research in cognition: how can we know how to characterize internal representations and processes. The solution is the same solution as in other sciences: theories make different predictions. Here, theories of linguistic reasoning make different predictions from theories of imaginal reasoning for patterns of processing times, errors, similarity judgments and more. Consider memory for schematic faces and names differing in visual features of the faces and linguistic features of the names. When the task encourages visual encoding, items that are visually similar are more confusable but when the task encourages linguistic encoding, items that have similar names are more confusable (Tversky, 1969).

Imaginal and linguistic encoding are characteristics of mental representations. Other tasks demonstrate imaginal characteristics of thinking, of transformations of mental representations. Consider the following problem: Alan is taller than Bruce. Charley is shorter than Bruce. Is Charley taller than Bruce? Dozens of experiments on this sort of task suggest that people solve them by lining up the elements on an imaginary line and comparing them. The primary evidence is that people are faster on far comparisons because they are easier to discriminate than on near ones. This effect, the Symbolic Distance Effect, appears for abstract comparisons, such as judgments of pleasantness or intelligence, as well as concrete comparisons, such as height, indicating that the underlying inference processes are the same (Banks & Flora, 1977; Paivio, 1978). Other studies have demonstrated that people solve simple syllogisms by constructing and transforming mental models (Johnson-Laird, 1983). Accuracy in solving the problems is determined by the number of mental models that must be kept in mind rather than linguistic characteristics of the syllogisms. This is especially impressive as syllogistic reasoning is one of the earliest examples of language-like reasoning. It may be that logicians solve problems in this way, but it appears that college students form mental models with spatial and visual characteristics to do so.
Studies of people’s mental representations of space show directly what aspects of space are captured and how they are used in reasoning. In one set of studies, students studied descriptions of environments, such as a small town or convention center (Taylor and Tversky, 1992). The descriptions were written from either a survey perspective in which landmarks were described relative to other landmarks in terms of north, south, east, and west or from a route perspective, in which landmarks were described relative to the changing position of a traveler in the environment, in terms of the traveler’s left, right, front, and back. Although students read only a single perspective, they answered true/false statements from both perspectives. The information in both descriptions was sufficient to answer the questions. The questions of interest required inferences, that is, identifying arrangements of landmarks from a new vantage point in the environment. If reasoning is linguistic, then there should be an advantage to questions in the same perspective as the description. However, if reasoning is based on spatial transformations of a mental representation of the space, the perspective of the question should not matter, and it did not. Students were equally fast and accurate to verify statements from the descriptions perspective as from the other perspective.

Other tasks show that spatial characteristics of bodies and the world account for the time it takes to retrieve locations of objects even when the entire task is verbal. As before, students study descriptions; in this case, the descriptions informed participants that they were in environments such as a museum or opera house with objects located to all six sides of their bodies. Then the descriptions informed participants that they had turned to face a new object. Participants were then queried about the objects now located to the different sides of the body (Tversky and Franklin, 1990). On the whole, participants found the task easy and their performance was nearly error free. There were, however, large differences in the time it took to retrieve objects, depending on characteristics of the body and the world. The body has three axes: the head/foot and front/back axes, which are asymmetric in both perception and behavior, and the left/right axis, which lacks salient asymmetries. Asymmetric axes are easier to discriminate, so identifying objects on those axes should be faster, and it is. In addition, the world has a single asymmetric axis, that created by gravity. For the canonically upright observer, the axis of gravity corresponds to the head/foot axis of the body, giving it an extra advantage imaginally, which translates into faster retrieval times for head/foot than front/back. Variations of the spatial situation yield predictable variations in the patterns of reaction time (Bryant, Tversky, and Franklin, 1992; Bryant and Tversky, 1999; Bryant, Tversky, and Lanca, 2001; Franklin, Tversky, and Coon, 1992; Tversky, Kim, and Cohen, 1999).

4. Conclusion: Imaginal Reasoning

Research on external representations as well as research on internal mental representations has supplied ample evidence that people spontaneously use both to represent spatial and non-spatial ideas and to reason with them. Spatial representations, whether external or internal, facilitate judgments and inferences that depend on spatial properties, such as direction, distance, configuration, and similarity. Because people have relied on those representations and inferences throughout their lives, to reach for things in space, to arrange and rearrange things in space, to navigate through space, they...
are especially adept at spatial representations and inferences. Thinking and reasoning in space, then, precedes language both phylogenetically and ontogenetically. It stands to reason that structures and concepts on language were built at least in part on structures and concepts used to perceive, understand, and act in the spatial world. People have a long history of thinking about abstract concepts spatially. Languages all over the world reflect this: we say we feel close to a friend, that a field is wide open, that our ideas are approaching each other, that a student is at the top of the class, that unemployment is falling. This is not to say that spatial representation and reasoning is always accurate. On the contrary, there are systematic errors and biases in both spatial representations and reasoning (e.g., Tversky, 2000; 2005a). For example, people judge two cities in the same geographic category to be closer than two cities in different geographic categories (e.g., Hirtle and Jonides, 1985). People also judge individuals in the same political or social group to be more similar than individuals in different groups (e.g., Quattrone, 1986). It is significant that the kinds of errors that appear in spatial reasoning also appear in abstract reasoning, additional evidence that abstract reasoning can bootstrap onto imaginal reasoning.
References


Bryant, D. J., & Tversky, B., 1999, Mental representations of spatial relations from diagrams and models. *Journal of Experimental Psychology: Learning, Memory and Cognition*, 25, 137-156.


